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# Control and synchronization of hyperchaos in digital manufacturing supply chain[☆]

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ABSTRACT

In this paper, we construct a mathematical model for a supply chain with computer aided digital manufacturing process. We show that the model exhibits various complicated behaviors including chaotic attractors. A stabilizing linear feedback controller is designed to stabilize the supply chain system. We move on to develop an active controller to synchronize two such systems. The design methodologies are based on Lyapunov stability theory. By simulating the model numerically, we observe the chaotic behaviours of the model. We find that the system has chaotic behaviours over a wide range of the values of the system parameters. The numerical results obtained demonstrate the effectiveness of the designed controllers. Our study reveals the interactions between the production and demand, in which the computer aided design plays an important role.



## 1. Introduction

Digital manufacturing has been introduced in industry to reduce the time of developing new products and the cost. Such computer aided design technology allows manufacturers to cater to the needs of customers, improve the quality of the products and adjusting product design according to market feedback. As a highly promising technology, the digital manufacturing process has been widely adopted in industry. It is thus important to investigate the effects of the computer aided manufacturing process on the production. Recently, a variety of digital manufacturing models have been designed [1–6]. In the 1960s and 1970s, the inventory control and material requirements planning (MRP) systems were introduced to

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the industry. In [7], Umble et al. integrated tools that are capable of providing capacity and sales planning functionalities, scheduling and forecasting into MRP. In order to make the production more competitive and efficient, process planning is introduced to guide the sequence of production process [8]. Controlling system based on computer program has been integrated into the process planning to create a consistent, processing procedure. Such a computer-aided process planning (CAPP) system is an essential component of CIM environment [9]. Kim and Duffie [10] designed a discrete time dynamic system to model the closed-loop process planning control and performed a thorough analysis. They show that such a control is capable of improving response to disturbances caused by rush orders and periodic fluctuations in capacity. In [11], Veda et al. proposed a simultaneous process planning and scheduling approach based on evolutionary artificial neural networks to solve the conflict between process planing and a production schedule. In [12], Baldwin et al. designed an enhancing simulation software for digital manufacturing systems.

Supply chain plays an important role in the production of automobile and aircraft. Investigations indicate that up to 60 percent of the value of these products comes from suppliers. Efficient computer aided digital manufacturing systems will help ensure continuous and efficient supply [13,14]. Over the past decade, the design, analysis and modeling of supply chain have been intensively studied in the literature. In [15], Hou et al. proposed an integrated system with production, inventory and distribution being integrated into the model in a two-stage supply chain network. In [16], Zhang et al. established a novel supply chain network model with nonlinear complementarity formulation and carried out qualitative analysis. In [17], Nikel et al. considered a multi-period stochastic supply chain network under financial decision and risk management. A three-phase multi-product supply chain network model was studied in [18], where a genetic algorithm method is used to solve the design problems in multi-product multi-phase supply chain network. “Bullwhip effect” and its consequence have been investigated in the literature. It is shown that such effects are responsible for chaotic behaviors of many supply chain models. Chaos theory has been used to study finance, economics and management [19]. In [20], Huang et al. investigated the bullwhip effect with stochastic control and $H_\infty$ control. In [21], Zhang et al. designed control laws to synchronize two chaotic supply chain systems with bullwhip effect by using radial basis function. In [22], Goksu et al. used active control method to achieve the synchronization of chaotic supply chain management systems.

A supply chain with digital manufacturing system may display complex dynamic behaviors and the control and synchronization of such a system may have potential applications in the management of real-word supply chain systems. In this paper, we design a supply chain model with computer aided design to investigate the dynamics of such a system. We design an active controller and a linear feedback controller to control the chaotic supply chain system and synchronize two supply chain systems, respectively. Numerical simulations are provided to illustrate the effectiveness and applicability of the proposed methods.

The reminder of the paper is organized as follows. In Section 2, we introduce a demand-driven supply chain model with computer aided manufacturing process. In Section 3, we design feed-back controllers to synchronize two such systems with different initial conditions. In Section 4, we consider the stabilization of the supply chain model. Conclusions are presented in Section 5.

## 2. Digital manufacturing supply chain modeling

In this section, we consider the design of a demand-driven supply chain model with computer aided digital manufacturing process. Recently, Anne et al. designed a supply chain model in [23]. In the following, we introduce the digital design process into the model and then consider the effects of the computer aided manufacturing process on the supply chain. The following notations are used in the supply chain modeling process [23].

$i$ Time period
$m$ Delivery efficiency of retailer
$n$ Rate of fulfilling the customer demand
$r$ Rate of information distortion for the demand by the customer for the products
$k$ Safety stock coefficient for the manufacturer
$x_i$ The demand quantity by the customer for the products in the current period
$y_i$ The supply quantity by the distributor in the current period

Here, we assume that a digital design process is required to be performed before the manufacturing. We thus introduce the following parameters and varables.

$c$ Conversion rate from digital design to product
$d$ The rate of digital design being produced
$z_i$ The quantity of finished computer aided digital designs
$w_i$ The quantity produced based on the digital design.

In the following, we consider the interaction among the quantity of products required by customer in the current period, the quantity of products that the distributors can supply in the current period, the quantity of finished computer aided digital designs, and the quantity of products being produced in response to the order in the current period. Assume that information is transmitted along the links of the supply chain with one unit of time delay and as such, the behavior of the model at time $i+1$ is based on the information at time period $i$. The information distortion caused by time delay will lead

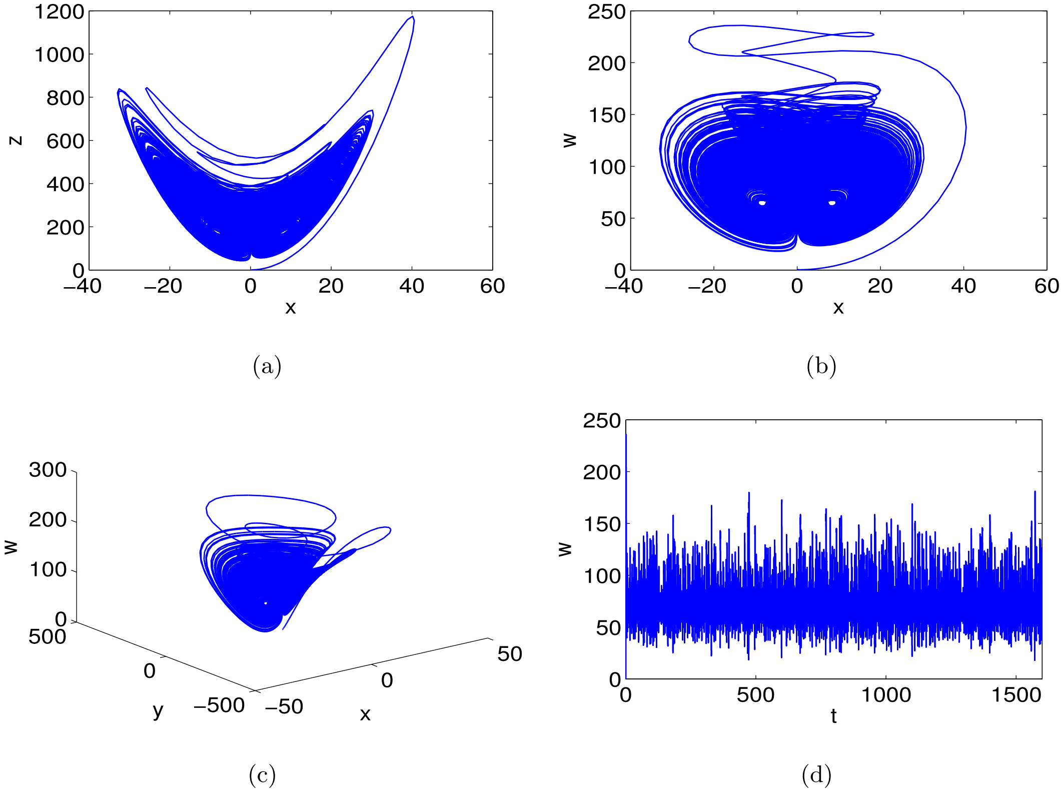



**Fig. 1.** Simulated phase portrait of system (2.5) when $m = 0.9$, $n = 0.8$, $r = 64.7$, $d = 0.7$, $c = 1.4$, $k = 4.8$, $\vartheta_m = 0.1$, $\vartheta_n = 0.2$, $\vartheta_r = 0.3$, $\vartheta_d = 0.1$, $\vartheta_c = 0.2$ and $\vartheta_k = 0.3$ with initial condition $x_0 = 0.2y_0 = 0.2$, $z_0 = 0.3$ and $w_0 = 0.32$: (a) projected onto the $x - z$ plane; (b) projected onto the $y - w$ plane; (c) in the $x - y - w$ space; (d) time history of $w$.

to complicated dynamic behavior. The interaction in the supply chain is illustrated in Fig. 1. We notice that the quantity of products required by customer in the current period is proportional to the quantity of products that the distributors can supply in the previous period. The current quantity of products required by customer is also influenced by the demand at the previous time period. Thus, we obtain the following difference equation

$$x_i = my_{i-1} - nx_{i-1}, \tag{2.1}$$

where $m$ is the delivery efficiency of the distributors and $n$ is the ratio of customer demand being fulfilled.

Next, we consider the current quantity of products that the distributors can supply. We assume that the distributors will consider the quantity of products required by customer and the quantity of products being produced. Then the corresponding interaction is described by

$$y_i = x_{i-1}(r - w_{i-1}), \tag{2.2}$$

where $r$ is the information distortion coefficient of the demand for the quantity of products by customer. Computer aided design has been widely used in manufacture. In this article, we introduce the process of digital design into the supply chain model. Before manufacturing a product, computer aided design will be performed. Then, based on the design, production will be carried out. Here, we use $z$ to denote the quantity of computer aided designs, which is proportional to the demand for the quantity of products by customer and the quantity of products that the distributors can supply in the previous period. Thus, we have

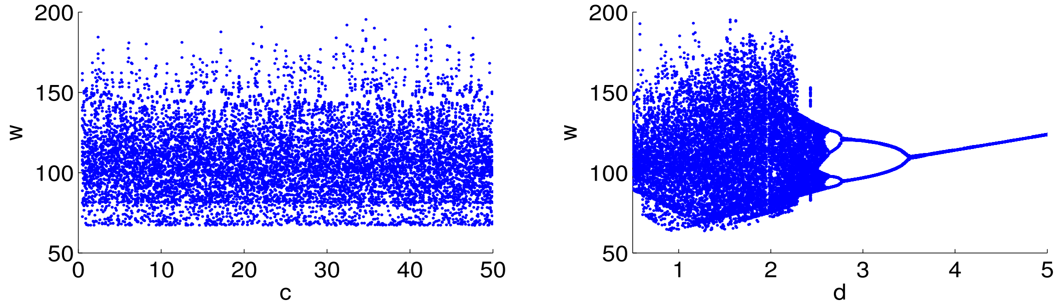
$$z_i = -dz_{i-1} + x_{i-1}y_{i-1} + z_{i-1}, \tag{2.3}$$

where $d$ is the rate that the design is put into production.

After the design, production will be carried out. The current quantity of products being produced is given by

$$w_i = cdz_{i-1} - kw_{i-1}, \tag{2.4}$$

where $c$ is the conversion rate.




(a) (b)

**Fig. 2.** Bifurcation diagrams of system (2.5). Except as described below, the values of the parameters are $m = 1$, $m = 0.9$, $n = 0.8$, $r = 64.7$, $d = 0.7$, $c = 1.4$, $k = 4.8$, $\vartheta_m = 0.1$, $\vartheta_n = 0.2$, $\vartheta_r = 0.3$, $\vartheta_d = 0.1$, $\vartheta_c = 0.2$ and $\vartheta_k = 0.3$ with initial condition $x_0 = 0.2y_0 = 0.2$, $z_0 = 0.3$ and $w_0 = 0.32$. (a) $c$ varies from 0.5 to 50. (b) $d$ varies from 0.5 to 5.

Based on the discrete time model (2.1)–(2.4), we obtain the model described by the system of differential equations as given below:

$$\begin{cases} x' = my - (n+1)x, \\ y' = rx - xw - y, \\ z' = -dz + xy, \\ w' = cdz - (k-1)w, \end{cases} \tag{2.5}$$

where $m$ is the delivery efficiency of retailers (1/days), $n$ is the rate of customer demand being fulfilled (1/days), $r$ is the rate of information distortion on the quantity of products required by customer (1/days), $k$ is the safety stock coefficient for manufacturer (1/days), $c$ is the conversion rate from digital design to product (dimensionless) and $d$ is the rate of digital design being produced (1/days).

Computer aided digital manufacturing process is widely used in production, which has a profound impact on the entire supply chain. In order to study the influence of such a process on the whole supply chain, we integrate the computer-aided digital manufacturing process into the supply chain to obtain system (2.5) based on the model proposed in [23].

External factors such as temperature and traffic condition have a significant influence on the dynamics of the supply chain. In the following, we integrate these perturbations into the supply chain model. Here, the uncertainties are modeled using parameters $\vartheta_m$, $\vartheta_n$, $\vartheta_r$, $\vartheta_k$, $\vartheta_c$ and $\vartheta_d$. Thus, we have

$$\begin{cases} x' = (m+\vartheta_m)y - (n+1+\vartheta_n)x + d_1, \\ y' = (r+\vartheta_r)x - xw - y + d_2, \\ z' = -(d+\vartheta_d)z + xy + d_3, \\ w' = (c+\vartheta_c)(d+\vartheta_d)z - (k-1-\vartheta_k)w + d_4, \end{cases} \tag{2.6}$$

where $\vartheta_m$, $\vartheta_n$, $\vartheta_r$, $\vartheta_k$, $\vartheta_d$ (1/days) and $\vartheta_c$ (dimensionless) are, respectively, the linear perturbations on the system parameters $m$, $n$, $r$ and $k$, and $d_1$, $d_2$, $d_3$, $d_4$ are nonlinear perturbations.

System (2.6) displays a complex dynamic behaviour. As shown in Fig. 1, when $m = 0.9$, $n = 0.8$, $r = 64.7$, $d = 0.7$, $c = 1.4$, $k = 4.8$, $\vartheta_m = 0.1$, $\vartheta_n = 0.2$, $\vartheta_r = 0.3$, $\vartheta_d = 0.1$, $\vartheta_c = 0.2$, $\vartheta_k = 0.3$, $d_1 = 0.3\sin t$, $d_2 = 0.2\cos 2t$, $d_3 = \sin 3t$, and $d_4 = \cos t$, the trajectory of system (2.6) is a chaotic attractor. Numerical analysis shows that the system behaves unpredictably and is highly sensitive to the initial condition. Such chaotic behaviors are caused by the interplays among the quantity of products required by customer, the quantity of products that the distributors can supply, the quantity of finished computer aided digital designs, and the quantity of products being produced in response to the order. The time delay of transmission of information is also one of the reasons causing complex dynamics.

System (2.6) has complex dynamic behaviours over a wide range of values of the system parameters. Here, we are particularly interested in the computer aided digital manufacturing process. We create bifurcation diagrams with the variation of $c$ and $d$ (see Fig. 2). We notice that the system behave chaotically as $c$ varies from 0.5 to 50. Fig. 2 (b) indicates that, when $d \in [0.5, 2.55]$, the system has chaotic behaviors and for $d > 2.55$, the chaotic behavior vanishes.

We find that, when the conversion parameters $c$ and $b$ are in a specific numerical range, there is chaos in the inventory system; when the conversion parameters are greater than the upper bound of this range, the chaos disappears. This phenomenon is consistent with the actual situation. In fact, when the conversion rate is large, it means that goods flow fast, and hence the demand of consumers can be met immediately, so that the inventory system is stable. On the contrary, when the conversion rate is low, the demand of products by customers cannot be met in time, and hence consumers may seek other substitutes. As a consequence, it will lead to the overstock of the inventory, causing the system be become chaotic and unstable state.

In this article, we are particularly interested in the impact of the digital design process on the entire supply chain. Therefore, we choose the values of the parameters $c$ and $d$ to create the bifurcation diagram. The parameter $c$ is referred to as the conversion rate from digital design to production. The parameter $d$ is the rate of digital design being produced. The bifurcation diagram displays the complex dynamical behavior of the system. From Fig. 2(b), we can see that when $d$ is small, the behavior of the system is chaotic. With the increase of $d$, the chaotic behavior begins to disappear, and the behavior of the system tends to stable, implying that increasing the rate of digital design being produced can stabilize the supply chain system. From Fig. 2(a), it is observed that the dynamic behavior of the system is chaotic for a wide range of values of $c$. The system cannot be stabilized through adjusting the value of the parameter $c$.

## 3. Synchronization of hyperchaos in digital manufacturing supply chain system

### 3.1. Synchronization

In this section, we consider the synchronization of two supply chain systems with different initial conditions. Since the supply chain behaves chaotically, the two systems are both sensitive to the initial conditions. Clearly, the trajectories of the two systems behave differently without an appropriate controller being applied to these systems. For some real life situations, it is sometime required that several supply chains to have the same behaviour. This can be realized using chaos synchronization strategy. The two chaotic supply chain models are named as the drive supply chain system (denoted by subscript 1) and the response supply chain system (denoted by subscript 2). They are, respectively, defined as follows:

$$\begin{cases} x_1' = (m+\vartheta_m)y_1 - (n+1+\vartheta_n)x_1 + 0.3\sin(t), \\ y_1' = (r+\vartheta_r)x_1 - y_1 - x_1w_1 + 0.2\cos(2t), \\ z_1' = -(d+\vartheta_d)z_1 + x_1y_1 + \sin(3t), \\ w_1' = (c+\vartheta_c)(d+\vartheta_d)z_1 - (k-1-\vartheta_k)w_1 + \cos(t), \end{cases} \tag{3.1}$$

and

$$\begin{cases} x_2' = (m+\vartheta_m)y_2 - (n+1+\vartheta_n)x_2 + 0.3\sin(t) + u_1(t), \\ y_2' = (r+\vartheta_r)x_2 - y_2 - x_2w_2 + 0.2\cos(2t) + u_2(t), \\ z_2' = -(d+\vartheta_d)z_2 + x_2y_2 + \sin(3t) + u_3(t), \\ w_2' = (c+\vartheta_c)(d+\vartheta_d)z_2 - (k-1-\vartheta_k)w_2 + \cos(t) + u_4(t), \end{cases} \tag{3.2}$$

where $u_1(t)$, $u_2(t)$, $u_3(t)$, $u_4(t)$ are the control functions to be determined. In order to evaluate the synchronization between the two systems, the drive supply chain system is subtracted from the response supply chain system to obtain

$$\begin{aligned} e_1 &= x_2 - x_1, \\ e_2 &= y_2 - y_1, \\ e_3 &= z_2 - z_1, \\ e_4 &= w_2 - w_1, \end{aligned} \tag{3.3}$$

where $e_1$, $e_2$, $e_3$ and $e_4$ represent errors between the drive system and the response system. Then,

$$\begin{cases} e_1' = (m+\vartheta_m)e_2 - (n+1+\vartheta_n)e_1 + u_1(t), \\ e_2' = (r+\vartheta_r)e_1 - e_2 - x_2w_2 + x_1w_1 + u_2(t), \\ e_3' = -(d+\vartheta_d)e_3 + x_2y_2 - x_1y_1 + u_3(t), \\ e_4' = (c+\vartheta_c)(d+\vartheta_d)e_3 - (k-1-\vartheta_k)e_4 + u_4(t). \end{cases} \tag{3.4}$$

Eq. (3.4) is called the error system. Here, our aim is to ensure that the error system is asymptotically stable at the origin. The choice for $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$ is not unique. In this article, we choose

$$\begin{cases} u_1(t) = -(m+\vartheta_m)e_2 + (n+\vartheta_n)e_1, \\ u_2(t) = x_2w_2 - x_1w_1 - (r+\vartheta_r)e_1, \\ u_3(t) = -x_2y_2 + x_1y_1 + (d-1+\vartheta_d)e_3, \\ u_4(t) = -(c+\vartheta_c)(d+\vartheta_d)e_3 + (k-2-\vartheta_k)e_4. \end{cases} \tag{3.5}$$

Then, the error system (3.4) becomes

$$\begin{cases} e_1' = -e_1, \\ e_2' = -e_2, \\ e_3' = -e_3, \\ e_4' = -e_4. \end{cases} \tag{3.6}$$

Now, the Lyapunov function for the error system (3.6) is designed as

$$V(e_1, e_2, e_3, e_4) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2). \tag{3.7}$$

Obviously, the Lyapunov function (3.7) is a positive definite function and it is equal to zero at the equilibrium of error system (3.6). The time derivative of Lyapunov function $V$ is

$$V'(t) = e_1'e_1 + e_2'e_2 + e_3'e_3 + e_4'e_4 = -(e_1^2 + e_2^2 + e_3^2 + e_4^2). \tag{3.8}$$

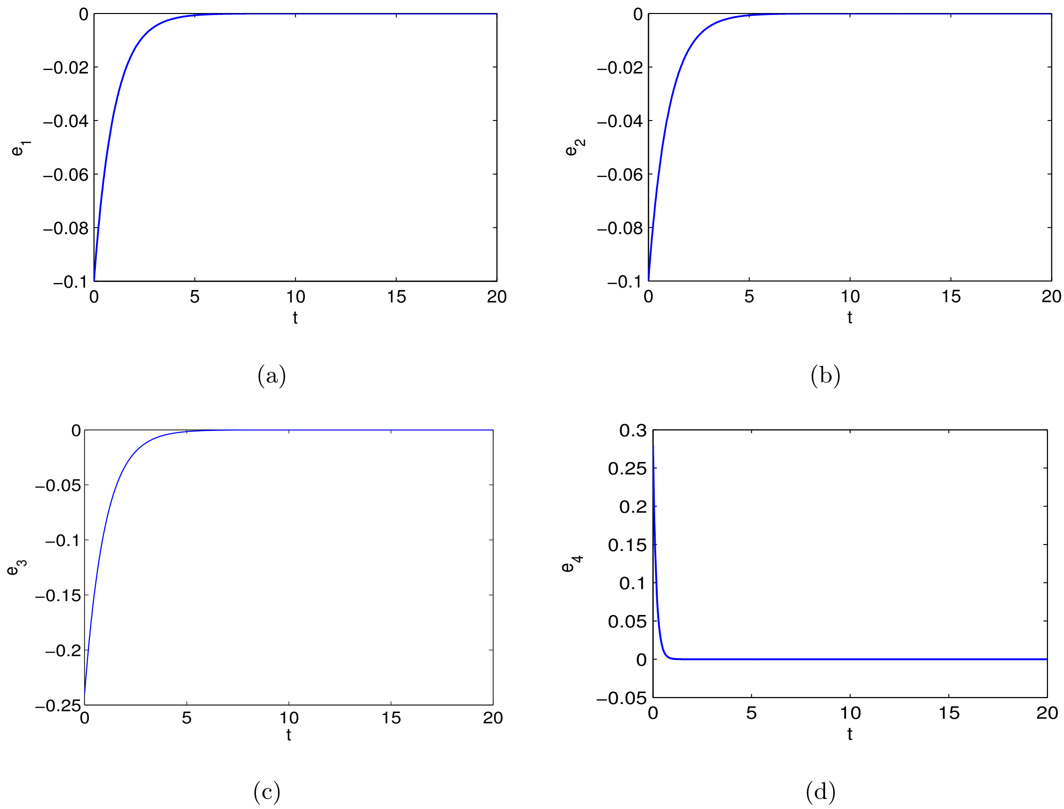



**Fig. 3.** Time history of error system (3.4) when $m = 0.9$, $n = 0.8$, $r = 64.7$, $d = 0.7$, $c = 1.4$, $k = 4.8$, $\vartheta_m = 0.1$, $\vartheta_n = 0.2$, $\vartheta_r = 0.3$, $\vartheta_d = 0.1$, $\vartheta_c = 0.2$ and $\vartheta_k = 0.3$ with initial conditions $x_{10} = 0.2$, $y_{10} = 0.2$, $z_{10} = 0.3$, $w_{10} = 0.32$, $x_{20} = 0.1$, $y_{20} = 0.1$, $z_{20} = 0.06$, $w_{20} = 0.6$.

Since $V'(t)$ is negative definite, by Lyapunov direct method, the zero solution of (11) is asymptotically stable. This implies that the drive system and response system are synchronized.

### *3.2. Simulation result*

In this section, we consider two supply chains with different initial conditions. As shown in Fig. 3, trajectory of the error system approaches to zero for different initial conditions. This implies that the two chaotic systems are synchronized, showing that the proposed control technique is effective. Therefore, although there are some destructive factors such as bullwhip effect, the inventory synchronization enables enterprises to adapt so as to effectively reduce risk.

Another way to verify the synchronization is to compare the time histories of the drive supply chain system and the response supply chain system. Below, we plot the time histories of the drive supply chain system (3.1) and the response supply chain system (3.2) in the same figure to verify that the two systems are synchronized. In Fig. 4, the red line represents the time history of drive supply chain system with initial condition ($x_{10} =1$, $y_{10} = 2$, $z_{10} = 3$, $w_{10} = 3$). The blue line represents the time history of the response supply chain system with initial condition ($x_{20} = 20$, $y_{20} = 30$, $z_{20} = 160$, $w_{20} = 80$). As shown in Fig. 4, the initial conditions of the drive supply chain system and the response supply chain system are different. Over time, the time histories of the two systems are on top of each other, suggesting that the two systems are synchronized, i.e., the designed controllers are effective.

## 4. Control of hyperchaos in digital manufacturing supply chain system

### *4.1. Chaos control of the supply chain model*

In this section, we consider the control of system (2.6). A linear feedback control will be applied to the supply chain model (2.6) to eliminate the hyperchaos. The controlled supply chain system can be written as

$$\begin{cases} x' = (m + \vartheta_m)y - (n + 1 + \vartheta_n)x + d_1 + u_1, \\ y' = (r + \vartheta_r)x - xw - y + d_2 + u_2, \\ z' = -(d + \vartheta_d)z + xy + d_3 + u_3, \\ w' = (c + \vartheta_c)(d + \vartheta_d)z - (k - 1 - \vartheta_k)w + d_4 + u_4, \end{cases} \tag{4.1}$$

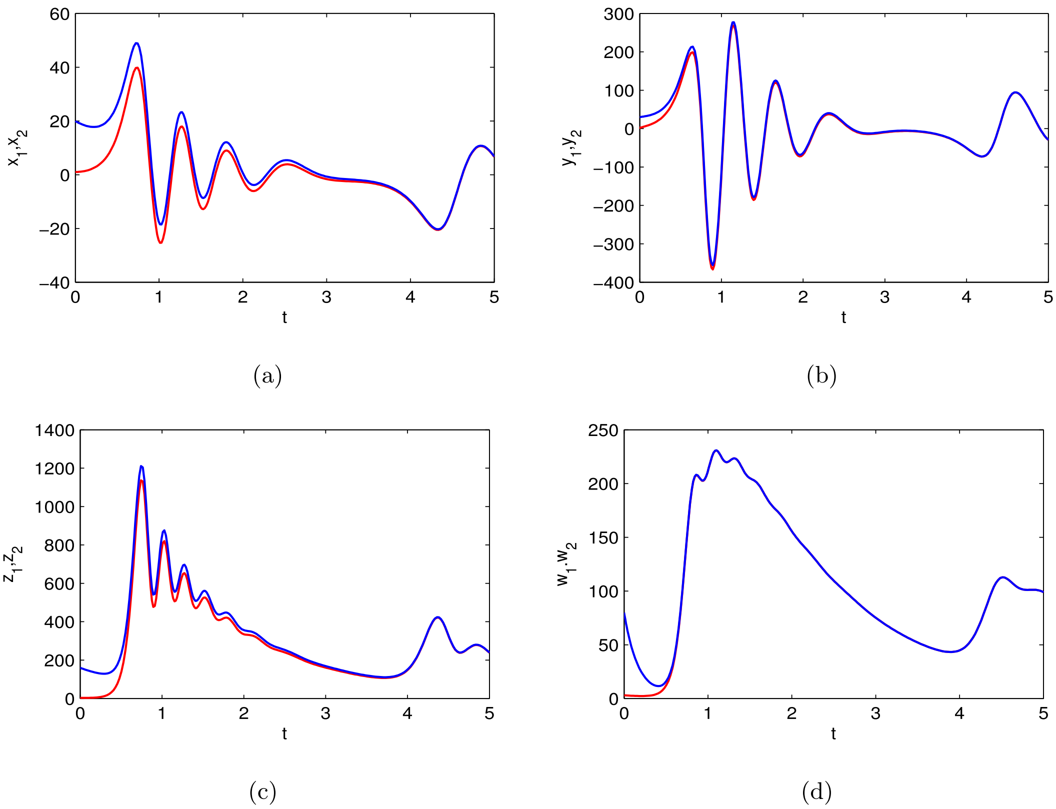



**Fig. 4.** Time histories of system (3.1) (red line) and system (3.2) (blue line) when $m = 0.9,\ n = 0.8,\ r = 64.7,\ d = 0.7,\ c = 1.4,\ k = 4.8,\ \vartheta_m = 0.1,\ \vartheta_n = 0.2,\ \vartheta_r = 0.3,\ \vartheta_d = 0.1,\ \vartheta_c = 0.2$ and $\vartheta_k = 0.3$ with initial conditions $x_{10} = 1,\ y_{10} = 2,\ z_{10} = 3,\ w_{10} = 3,\ x_{20} = 20,\ y_{20} = 30,\ z_{20} = 160,\ w_{20} = 80$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

where $u_1,\ u_2,\ u_3,\ u_4$ are the controllers to be designed.

Let $\bar{x},\ \bar{y},\ \bar{z},\ \bar{w}$ be the stationary state values of the controlled supply chain model (4.1) for $d_1 = d_2 = d_3 = d_4 = 0$. Define $\hat{x} = x - \bar{x},\ \hat{y} = y - \bar{y},\ \hat{z} = z - \bar{z},\ \hat{w} = w - \bar{w}$. The feedback controllers are designed as follows:

$$\begin{aligned} u_1 &= -k_1\hat{x}, \\ u_2 &= -k_2\hat{y}, \\ u_3 &= -k_3\hat{z}, \\ u_4 &= -k_4\hat{w}, \end{aligned} \tag{4.2}$$

where $k_1,\ k_2,\ k_3,\ k_4$ are positive feedback gains. Applying the proposed controller to the supply chain model yields

$$\begin{cases} x' = (m + \vartheta_m)y - (n + 1 + \vartheta_n)x - k_1(x - \bar{x}), \\ y' = (r + \vartheta_r)x - xw - y - k_2(y - \bar{y}), \\ z' = -(d + \vartheta_d)z + xy - k_3(z - \bar{z}), \\ w' = (c + \vartheta_c)(d + \vartheta_d)z - (k - 1 - \vartheta_k)w - k_4(w - \bar{w}). \end{cases} \tag{4.3}$$

We notice that (0,0,0,0) is the equilibrium point of Eq. (4.3). When the zero equilibrium of Eq. (4.3) is asymptotically stable, $xw$ and $xy$ are zero. Then, Eq. (4.3) can be written as

$$\begin{cases} x' = (m + \vartheta_m)y - (n + 1 + \vartheta_n)x - k_1 x, \\ y' = (r + \vartheta_r)x - y - k_2 y, \\ z' = -(d + \vartheta_d)z - k_3 z, \\ w' = (c + \vartheta_c)(d + \vartheta_d)z - (k - 1 - \vartheta_k)w - k_4 w. \end{cases} \tag{4.4}$$

The Jacobi matrix of system (4.4) at the zero equilibrium is

$$J = \begin{pmatrix} -(n + 1 + \vartheta_n + k_1) & m + \vartheta_m & 0 & 0 \\ r + \vartheta_r & -(1 + k_2) & 0 & 0 \\ 0 & 0 & -(d + \vartheta_d + k_3) & 0 \\ 0 & 0 & (c + \vartheta_c)(d + \vartheta_d) & -(k - 1 - \vartheta_k + k_4) \end{pmatrix}. \tag{4.5}$$

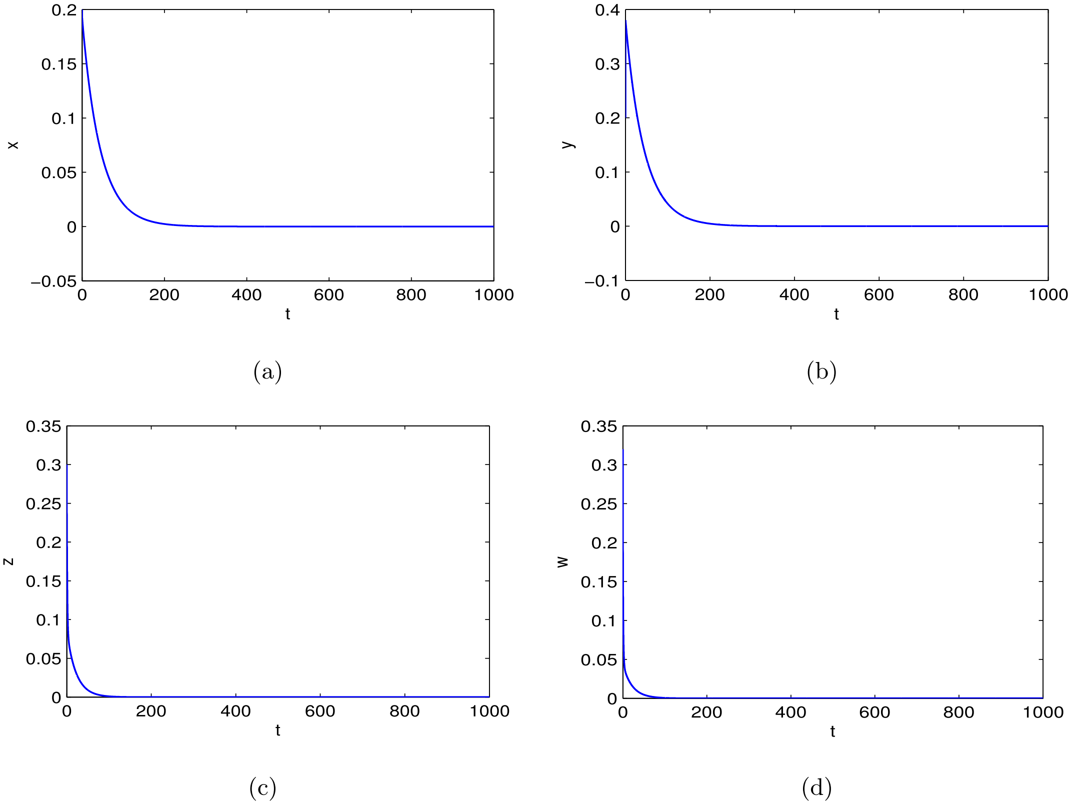



**Fig. 5.** Time history of error system (4.3) when $m = 0.9,\ n = 0.8,\ r = 64.7,\ d = 0.7,\ c = 1.4,\ k = 4.8,\ \vartheta_m = 0.1,\ \vartheta_n = 0.2,\ \vartheta_r = 0.3,\ \vartheta_d = 0.1,\ \vartheta_c = 0.2,\ \vartheta_k = 0.3,\ k_1 = 0,\ k_3 = 0$ and $k_4 = 0$ with initial condition $x_0 = 0.2\ y_0 = 0.2,\ z_0 = 0.3$ and $w_0 = 0.32$.

The characteristic equation of matrix (4.5) is

$$(\lambda+d+\vartheta_d+k_3)(\lambda+k-1-\vartheta_k+k_4)[(\lambda+n+1+\vartheta_n+k_1)(\lambda+1+k_2)-(m+\vartheta_m)(r+\vartheta_r)]=0. \tag{4.6}$$

When the values of the parameters are $m = 0.9,\ n = 0.8,\ r = 64.7,\ d = 0.7,\ c = 1.4,\ k = 4.8,\ \vartheta_m = 0.1,\ \vartheta_n = 0.2,\ \vartheta_r = 0.3,\ \vartheta_d = 0.1,\ \vartheta_c = 0.2,\ \vartheta_k = 0.3$, we assume that the values of the control parameters are $k_1 = 0,\ k_3 = 0$, and $k_4 = 0$. Then the characteristic equation of Eq. (4.6) is

$$(\lambda + 0.8)(\lambda + 3.5)[\lambda^2 + (k_2 + 3)\lambda + 2k_2 - 63] = 0. \tag{4.7}$$

According to the Routh-Hurwitz criterion, all real eigenvalues and all real parts of complex conjugate eigenvalues are negative. From (4.7), we know that

$$\lambda_1 = -0.8 < 0,\ \ \lambda_2 = -3.5 < 0.$$

If $k_2 + 3 > 0,\ 2k_2 - 63 > 0$, then $\lambda_3 < 0,\ \lambda_4 < 0$. Hence, $k_2 > \frac{63}{2}$.

Therefore, the zero equilibrium of the controlled digital manufacturing supply chain system (4.1) is asymptotically stable under the control with the values of its parameters being given by $k_1 = 0,\ k_2 > \frac{63}{2},\ k_3 = 0$, and $k_4 = 0$.

### *4.2. Simulation*

Now, numerical simulation will be carried out to illustrate the applicability of the controller designed in earlier section. As shown in Fig. 5, by choosing $k_1 = 0,\ k_2 = 32,\ k_3 = 0$ and $k_4 = 0$, the trajectory of system (4.3) with initial condition $x_0 = 0.2 y_0 = 0.2,\ z_0 = 0.3$ and $w_0 = 0.32$ is no longer chaotic but convergent to the equilibrium. The simulation result shows the effectiveness of the designed control law. Therefore, we see that the designed control law can eliminate destructive behaviours such as bullwhip effect of the model, and giving rise to a stable supply chain inventory system.

## 5. Conclusion

In this paper, we construct a mathematical model for a supply chain system with computer aided digital design. We show that such a system displays complex dynamical behaviour including chaos in a wide range of the values of the system parameters. We create bifurcation diagrams to show that the rate at which the digital design is put into manufacturing has a significant influence on the dynamic behaviour of the model. We design controllers to synchronize two supply chain systems with different initial conditions. We also investigate the stabilization of the system. By designing effective controllers, the chaotic behavior of the model disappears and the trajectory converges to the equilibrium. Mathematical proofs and numerical simulations are both provided to show the correctness and effectiveness of the designed model. Our results has potential applications in the study of food chains with digital computer aided design.

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